

Contraction and decomposition matrices for vacuum diagrams

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Abstract

Tensor reduction of vacuum diagrams uses contraction and decomposition matrices. We present general recurrence relations for the calculation of those matrices and an explicit formula for the 3-loop decomposition matrix and its determinant.

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1 Introduction

This letter is mainly conceived as an extension of the two-loop results for contraction and decomposition matrices of [1]. For a list of many usefull applications and other literature on this matter we gladly refer the reader to this paper and the references therein. Here we would only like to point out that the higher loop results are also very important. For instance they can be applied in the calculations of the anomalous dimensions (currently 3 and 4 loops are feasible [2, 3]) or the moments of deep inelastic structure functions (cfr. 3-loop calculation in [4]).

The remainder of the article is organized as follows. In section 2 we introduce the notation. Although we tried to follow the notations of [1] as closely as possible, we had to make some adaptations to be able to describe the tensor structure of a general L -loop diagram. In section 3 the general recurrence relations are presented and in section 4 an explicit solution in the 3-loop case is derived. The last section is a summary and conclusion.

2 Notations

In general a L -loops diagram has N_i tensorindices in every loop $i = 1, \dots, L$. This gives us the index-set $I = \bigcup_i I_i$, $I_i = \{\mu_{i,1}, \mu_{i,2}, \dots, \mu_{i,N_i}\}$ for every loop i . A vacuum diagram $G[I]$ with this tensorstructure can be decomposed in all the possible products of metric tensors we can produce from the index-set I , i.e. in a shorthand we get

$$G[I] = \sum_{\sigma(I)} a_{\sigma} \prod_{j=1}^{N/2} g_{\sigma(j)\sigma(j+1)} \quad (1)$$

We always assume the $N = \sum_i N_i$ is even otherwise the vacuumdiagram $G[I]$ is identically zero. Many of these coefficients a will have the same value: the left-hand side of the equation is symmetrical in the indices of I_i , thus so should the right-hand side. Now if we consider the metric tensor to be an object which makes a connection, either between two different loops ($g_{\mu_{i,j}\mu_{j,k}}$) or within the same loop ($g_{\mu_{i,j}\mu_{i,k}}$), it is easy to see that the only distinct values a_{σ} will correspond to a different number of connections between the loops. Therefore we introduce an object which we call a *link* and contains this information. It is characterized by the number of connections t_{ij} between loop i en j ($L(L+1)/2$ numbers). It might also be characterized by the number of metric tensors s_i which stay within a certain loop (L

numbers). We have the following relations

$$2s_i + \sum_j t_{ij} = N_i, \forall i \in 1, 2, \dots, L. \quad (2)$$

The tensorconfiguration S_l is the symmetric sum of products of metric tensors which belong to a certain link l . The number of terms in such a tensorconfiguration is

$$c_l = \frac{\prod N_i!}{\prod (2s_i)! t_{ij}!}. \quad (3)$$

Now we can write (1) as

$$G[I] = \sum_l a_l S_l \quad (4)$$

where sum runs over all possible links for the tensorconfiguration I . We get a system of equations by contracting this expression with each $S_{l'}$:

$$G[I] \otimes S_{l'} = G[I]^{(l')} = \sum_l a_l S_l \otimes S_{l'} = \sum_l a_l \chi_{ll'} \quad (5)$$

which is the definition of the contraction matrix χ . Note that we write an \otimes if tensors are involved. This is can be inverted to

$$a_l = (\chi^{-1})_{ll'} G[I]^{(l')} = \phi_{ll'} G[I]^{(l')}. \quad (6)$$

where ϕ is the decomposition matrix. In this definition both matrices are symmetric. We use the notation $\chi_{ll'}$ and $\phi_{ll'}$ if the link is completely general. Otherwise we shall write

$$\begin{pmatrix} s \\ t \end{pmatrix} \chi \begin{pmatrix} s' \\ t' \end{pmatrix}. \quad (7)$$

Here s is the columnmatrix containing the number s_i of metric tensors which stay within the loop i and t is the triangular matrix containing the number t_{ij} of metric tensors which connect loops i and j . Here we will not write the corresponding indices though. So if we write (s) , we really mean (s_k) and $(s - \delta_i)$ is really $(s_k - \delta_{ik})$, k running from 1 to L and analogous for (t) .

In order to get used to this notations we will derive a simple identity which we will use further on. The explicit solution of this problem for the one-loop case was already known in [5]:

$$\begin{pmatrix} N/2 \end{pmatrix} \chi \begin{pmatrix} N/2 \end{pmatrix} = (N-1)!! 2^{N/2} (d/2)_{N/2} \quad (8)$$

Now if we have a general L -loop diagram and sum over the tensorconfigurations $\sum_l S_l$ this equals the tensorconfiguration of the 1-loop diagram with $I^{1\text{-loop}} = \cup_i I_i$. If r_l is an arbitrary term form S_l , we have using the symmetry of χ

$$\begin{aligned}\sum_{l'} \chi_{ll'} &= S_l \otimes \sum_{l'} S_{l'} = c_l r_l \otimes \sum_{l'} S_{l'} \\ &= \frac{c_l}{(N-1)!!} \sum_l S_l \otimes \sum_{l'} S_{l'} = \frac{c_l}{(N-1)!!} S_{1\text{-loop}} \otimes S_{1\text{-loop}} \\ &= c_l 2^{N/2} (d/2)_{N/2}\end{aligned}\quad (9)$$

using (8). If we multiply by ϕ , we obtain

$$\sum_{l'} \phi_{ll'} c_{l'} = \frac{1}{2^{N/2} (d/2)_{N/2}} \quad (10)$$

Here we have derived this expression on very general grounds, while the authors of [1] have proved it by explicit 2-loop calculations. Like these authors we will use identity (10) in order to obtain explicit solutions for the recurrence relations we will now derive.

3 Recurrence relations

Now we can construct recurrence relations for a contraction tensor. We can do this using a similar method as in [1] by writing S_l as a partial derivative of tensors. However we found it to be very convient to add the metric tensors one by one in $S_l \otimes S_{l'} = \chi_{ll'}$ in a grafic representation [6]. Either way we get the following recurrence relations for an L -loop contraction tensor

$$\begin{aligned}\begin{matrix} (s) \\ (t) \end{matrix} \chi_{(t')}^{(s')} &= \frac{N_i N_j}{t'_{ij}} \left[(d + N_i + N_j - t_{ij} - 1) \begin{matrix} (s) \\ (t-\delta_{ij}) \end{matrix} \chi_{(t'-\delta_{ij})}^{(s')} \right. \\ &\quad + t_{ij} \begin{matrix} (s-\delta_i-\delta_j) \\ (t+\delta_{ij}) \end{matrix} \chi_{(t'-\delta_{ij})}^{(s')} + \sum_{k \neq i,j} t_{ik} \begin{matrix} (s-\delta_i) \\ (t-\delta_{jk}+\delta_{ik}) \end{matrix} \chi_{(t'-\delta_{ij})}^{(s')} \\ &\quad + \sum_{k \neq i,j} t_{jk} \begin{matrix} (s-\delta_j) \\ (t-\delta_{ik}+\delta_{jk}) \end{matrix} \chi_{(t'-\delta_{ij})}^{(s')} + \sum_{k \neq i} \sum_{l \neq j,k} t_{kl} \begin{matrix} (s) \\ (t-\delta_{ik}-\delta_{jk}+\delta_{kl}) \end{matrix} \chi_{(t'-\delta_{ij})}^{(s')} \\ &\quad \left. + \sum_{k \neq i,j} 2s_k \begin{matrix} (s+\delta_k) \\ (t-\delta_{ik}-\delta_{jk}) \end{matrix} \chi_{(t'-\delta_{ij})}^{(s')} \right], \quad (11)\end{aligned}$$

and

$$\begin{aligned} \binom{(s)}{(t)} \chi \binom{(s')}{(t')} &= \frac{N_i(N_i - 1)}{2s'_i} \left[(d + 2N_i - 2s_i - 2) \binom{(s-\delta_i)}{(t)} \chi \binom{(s'-\delta_i)}{(t')} \right. \\ &\quad \left. + \sum_{k \neq i} \sum_{l < k} 2t_{kl} \binom{(s)}{(t-\delta_{ik}-\delta_{jk}+\delta_{kl})} \chi \binom{(s'-\delta_i)}{(t')} + \sum_{k \neq i} 2s_k \binom{(s+\delta_k)}{(t-2\delta_{ik})} \chi \binom{(s'-\delta_i)}{(t')} \right] \quad (12) \end{aligned}$$

These expressions for χ induce similar relations for ϕ . We get

$$\begin{aligned} t'_{ij} \binom{(s)}{(t-\delta_{ij})} \phi \binom{(s')}{(t'-\delta_{ij})} &= N_i N_j \left[(d - 1 + N_i + N_j - t_{ij}) \binom{(s)}{(t)} \phi \binom{(s')}{(t')} \right. \\ &\quad + (t_{ij} - 1) \binom{(s+\delta_i+\delta_j)}{(t-2\delta_{ij})} \phi \binom{(s')}{(t')} + \sum_{k \neq i,j} t_{ik} \binom{(s+\delta_i)}{(t+\delta_{jk}-\delta_{ik}-\delta_{ij})} \phi \binom{(s')}{(t')} \\ &\quad + \sum_{k \neq i,j} t_{jk} \binom{(s+\delta_j)}{(t+\delta_{ik}-\delta_{jk}-\delta_{ij})} \phi \binom{(s')}{(t')} \\ &\quad + \sum_{k \neq i} \sum_{l \neq j,k} t_{kl} \binom{(t+\delta_{ik}+\delta_{jk}-\delta_{kl}-\delta_{ij})}{(s)} \phi \binom{(s')}{(t')} \\ &\quad \left. + \sum_{k \neq i,j} 2s_k \binom{(s+\delta_k)}{(t-\delta_{ik}-\delta_{jk})} \phi \binom{(s')}{(t')} \right] \quad (13) \end{aligned}$$

and

$$\begin{aligned} (2s_i) \binom{(s-\delta_i)}{(t)} \phi \binom{(s'-\delta_i)}{(t')} &= N_i(N_i - 1) \left[(d + 2N_i - 2s_i - 2) \binom{(s)}{(t)} \phi \binom{(s')}{(t')} \right. \\ &\quad \left. + \sum_{k \neq i} \sum_{l < k} 2t_{ik} \binom{(s-\delta_i)}{(t+\delta_{ik}+\delta_{jk}-\delta_{kl})} \phi \binom{(s')}{(t')} + \sum_{k \neq i} 2s_k \binom{(s-\delta_k-\delta_i)}{(t+2\delta_{ik})} \phi \binom{(s')}{(t')} \right]. \quad (14) \end{aligned}$$

The expressions (11) and (12) are very usefull when contrsucting the matrices through recurrence relations, while (14) and (13) are not fit for direct use: they will however allow us to construct explicit solutions for the 3-loop case. We have tested these recurrence relations by comparing their results with a **Mathematica**-package by Misiak [7] which uses the actual $S_l \otimes S_{l'}$ contraction.

4 Explicit solutions

Like in [1] we will try to reduce the decomposition matrix to a unique value, which we will then compute by using relation (10). In the 3-loop cases



Figure 1: The different 3-loop cases

however, we have two classes, each of them with different endpoints for recurrence relations. They are shown in figure 1 (lines that connect vertices are metric tensors, dashed lines separate the loops)

We start from the simple case where $N_i \geq N_j + N_k$ and $s'_j = s'_k = t'_{jk} = 0$. By applying (14) on s_i we obtain the following endpoint of the recurrence relations: $t'_{ij} = t_{ij} = N_j$ and $t'_{ik} = t_{ik} = N_k$:

$$\begin{aligned}
& \frac{((N_i - N_j - N_k)/2, 0, 0)}{(0, N_j, N_k)} \phi_{(t)}^{(s)} \\
&= \frac{(-)^{s_i + (N_i - N_j - N_k)/2} s_i! (N_j + N_k)! (d/2 - 1 + N_j + N_k)}{N_i! (d/2 - 1 + N/2 - s_i)_{s_i+1}} \\
& \times \frac{(0, 0, 0)}{(0, N_j, N_k)} \phi_{(0, N_j, N_k)}^{(0, 0, 0)}
\end{aligned} \tag{15}$$

Calculating the normalisation factor $\frac{(0, 0, 0)}{(0, N_j, N_k)} \phi_{(0, N_j, N_k)}^{(0, 0, 0)}$ with the aid of (10) gives

$$\sum_{s_i} \sum_{s_j} \sum_{s_k} \frac{((N_i - N_j - N_k)/2, 0, 0)}{(N_j, N_k, 0)} \phi_{(t)}^{(s)} \frac{N_i! N_j! N_k!}{2^{\sum s} s_i! s_j! s_k! t_{ij}! t_{ik}! t_{jk}!} = \frac{1}{2^N (d/2)_N} \tag{16}$$

Because (15) is only dependent on s_i we shall evaluate the other 2 summations first. If we add a factor $(N_i - 2s_i)!$ in the numerator we obtain

$$\sum_{s_j} \sum_{s_k} \frac{(N_i - 2s_i)! N_j! N_k!}{2^{s_j + s_k} s_j! s_k! t_{ij}! t_{ik}! t_{jk}!}.$$

We recognize the expression (3) for counting the number of terms in a tensorconfiguration S_l with the following number of tensorindices $(N_i -$

$2s_i, N_j, N_k$). By summing over all possible s_j and s_k we can readily see that we really obtain the number of terms in a 2-loop diagram with $(N_i - 2s_i, N_j + N_k)$ tensorindices. This gives us

$$\sum_{s_j} \sum_{s_k} \frac{(N_i - 2s_i)! N_j! N_k!}{2^{s_j+s_k} s_j! s_k! t_{ij}! t_{ik}! t_{jk}!} = \frac{(N_j + N_k)!}{2^{(N_j+N_k-N_i+2s_i)/2} \left(\frac{N_j+N_k-N_i+2s_i}{2}\right)!} \quad (17)$$

The remaining summation over s_i is identical to the expression we get by inserting (25) in (26) of [1] which can be summed up to a hypergeometric function. Eventually we get

$$\frac{(0,0,0)}{(N_j, N_k, 0)} \phi_{(N_j, N_k, 0)}^{(0,0,0)} = \frac{\left(\frac{d-2}{2}\right)_{N_j+N_k}}{(N_j + N_k)! (d-2)_{N_j+N_k} \left(\frac{d}{2}\right)_{N_j+N_k}} \quad (18)$$

Note that we have not explicitly used the fact that there are only 3 loops: this result is valid for every diagram with $N_1 \geq \sum_{i=2}^L N_i$.

If $N_i \leq N_j + N_k, \forall i$ again we will start from the simpler case $\frac{(0)}{(t_0)} \phi_{(t)}^{(s)}$, where t_0 stands for $t_{ij} = \frac{N_i+N_j-N_k}{2}, \forall i \neq j$. Applying the recurrence relation (14) in this case eventually we will end up in a situation where $\frac{(0)}{(t_0)} \phi_{(t)}^{(s)}$ is completely expressed as a function of one unique unknown factor $\frac{(0)}{(t_0)} \phi_{(t_0)}^{(0)}$, which thus fulfils the role of a normalisation factor. The result of the recurrence relation (14) is

$$\begin{aligned} \frac{(0)}{(t_0)} \phi_{(t)}^{(s)} &= \sum_{m=\max(0, s_i-s_j)}^{s_i} \frac{\sum_{n=\max(0, m-s_k)}^{\min(m, t_{jk})}}{s_i!} \frac{\sum_{p=\max(0, s_j-s_i-s_k+2m-n)}^{\min(s_j-s_i+m, t_{ik}+2m-n)}}{s_j!} \frac{\sum_{p=\max(0, s_j-s_i-s_k+2m-n)}^{\min(s_j-s_i+m, t_{ik}+2m-n)}}{s_k!} \\ &\quad \frac{(s_i-m)!(m-n)!n!}{(s_j-s_i+m-p)!p!} \frac{(s_k-sm+n-s_j+s_i+p)!}{(t_{jk}-n)!} \frac{(t_{ik}+2m-n)!}{(t_{ik}+2m-n-p)!} \frac{(t_{ij}+2s_i-2m+n+p)!}{(t_{ij}+s_i+s_j-s_k)!} \\ &\quad (d/2-1+N_i-s_i)_{s_i} (d/2-1+N_j-s_j+s_i-m)_{s_j-s_i+m} \\ &\quad (d/2-1+N_k-(s_k+s_j+s_i-2m+n+p))_{s_k+s_j+s_i-2m+n+p} \\ &\quad \frac{(0)}{(t_0)} \phi_{(t_0)}^{(0)}. \end{aligned} \quad (19)$$

In order to calculate the normalisation factor we have to sum up this expression à la (16), which gives us a nine-fold summation! It is clear that this is practically a dead-end street. Nevertheless by making connection with the simpler case $N_i \geq N_j + N_k$ we will be able to prove the following lemma:

$$\frac{(0)}{(t_0)} \phi_{(t_0)}^{(0)} \stackrel{?}{=} f(N_i, N_j, N_k) = \frac{(d-2)_{N/2} \left(\frac{d}{2}\right)_{N/2} \prod_i N_i! (d/2 + N_i - 1)_{N/2-N_i}}{\prod_i (N/2 - N_i)! (d/2 + N/2 - N_i - 1)_{N_i}}. \quad (20)$$

We will prove by induction. In the borderline case $N_i = N_j + N_k$ (20) reduces to (18), which has been proven. This is the starting point of our induction.

In order to proceed we must find a relation between $\phi_{(t-\delta_{ij})}^{(0)} \phi_{(t-\delta_{ij})}^{(0)}$ and $\phi_{(t)}^{(0)} \phi_{(t)}^{(0)}$ (we now no longer write the index 0). In order to establish this connection we use (13)

$$\begin{aligned} & t_{ij} \phi_{(t-\delta_{ij})}^{(0)} \phi_{(t-\delta_{ij})}^{(0)} \\ &= N_i N_j [(d + N_i + N_j - t_{ij} - 1) \phi_{(t)}^{(0)} \phi_{(t)}^{(0)} + (t_{ij} - 1) \phi_{(t-2\delta_{ij})}^{(\delta_i+\delta_j)} \phi_{(t)}^{(0)} \\ & \quad + t_{ik} \phi_{(t+\delta_{jk}-\delta_{ik}-\delta_{ij})}^{(\delta_i)} \phi_{(t)}^{(0)} + t_{jk} \phi_{(t+\delta_{ik}-\delta_{jk}-\delta_{ij})}^{(\delta_j)} \phi_{(t)}^{(0)}] . \end{aligned}$$

and sequentially (14)

$$\begin{aligned} \phi_{(t-\delta_{ij})}^{(0)} \phi_{(t-\delta_{ij})}^{(0)} &= \frac{N_i N_j}{t_{ij}} \left[(d + N_i + N_j - t_{ij} - 1) - \frac{(t_{ij} - 1)}{d/2 - 2 + N_i} \right. \\ & \quad + \frac{(t_{ij} - 1)t_{jk}(t_{ik} + 1)}{(d/2 - 2 + N_i)(d/2 - 2 + N_j)} - \frac{t_{ik}(t_{jk} + 1)}{d/2 - 2 + N_i} \\ & \quad \left. - \frac{t_{jk}(t_{ik} + 1)}{d/2 - 2 + N_j} \right] \phi_{(t)}^{(0)} \phi_{(t)}^{(0)}, \end{aligned} \quad (21)$$

$f(N_i, N_j, N_k)$ on the other hand satisfies the following recurrence relation

$$\begin{aligned} f(N_i - 1, N_j - 1, N_k) &= \frac{N_i N_j (d/2 - 2 + N/2 - N_k)}{(\frac{N_i + N_j - N_k}{2})(d/2 - 2 + N_i)(d/2 - 2 + N_j)} \\ & \quad \times (d/2 - 2 + N/2)(d/2 - 3 + N/2) f(N_i, N_j, N_k) \end{aligned} \quad (22)$$

By substituting $t_{ij} = \frac{N_i + N_j - N_k}{2}$, $\forall i \neq j$ it is easy to establish the equality of (21) and (22) so not only the lemma $\phi_{(t_0)}^{(0)} \phi_{(t_0)}^{(0)} \equiv f(N_i, N_j, N_k)$ is proven but also the hideous expressions (19).

If we start from the general case $\phi_{(t')}^{(s')} \phi_{(t)}^{(s)}$ we can first apply (14) in order to reduce s'_1, s'_2 and s'_3 to zero. This recurrence relation can be solved explicitly (see [1]). Together with (19) and (20) we have established an explicit form for $\phi_{(t')}^{(s')} \phi_{(t)}^{(s)}$. After careful substitutions we can write this in a symmetrical

form

$$\begin{aligned}
\binom{s'}{t'}\phi_{(t)}^{(s)} &= \sum_{l''} \sum_{l'''} \sum_{l^v} \frac{\prod [(-)^{s''+s_i^v} s_i! s_i'!]}{\prod [s_i'''! N_i! (s_i^v - s_i''')! (s_i'' - s_i''')!]} \frac{\prod t_{ij}'''}{\prod [(\frac{t_{ij}'' - t_{ij}}{2})! (\frac{t_{ij}^v - t_{ij}'}{2})!]} \\
&\times \frac{t_{jk}!}{(t_{jk} - s_i'' + s_i''')!} \frac{t_{jk}'!}{(t_{jk}' - s_i^v + s_i''')!} \frac{(t_{ik} + s_i'' - s_i''')!}{(t_{ik} + s_i'' - s_i''')!} \\
&\times \frac{(t_{ik} + s_i^v - s_i''')!}{(t_{ik} + s_i^v - s_i''')!} \frac{(t_{ij} + s_i'' - s_i''')!}{(t_{ij} + s_i'' - s_i''')!} \\
&\times \frac{(t_{ij} + s_i^v - s_i''')!}{t_{ij}'''} \frac{\prod (\frac{d-2}{2} + t_{ij}'')}{(d-2)_{\Sigma t_{ij}''} (d/2)_{\Sigma t_{ij}''}} \\
&\times \frac{(\frac{d-2}{2})_{(t_{ij}+t_{ik}+t_{ij}''+t_{ik}'')/2} (\frac{d-2}{2})_{(t_{ij}'+t_{jk}'+t_{ij}''+t_{jk}'')/2} (\frac{d-2}{2})_{(t_{ik}'+t_{jk}'+t_{ik}''+t_{jk}'')/2}}{(\frac{d}{2} + t_{ij}'' + t_{ik}'') s_i'' (\frac{d}{2} + t_{ij}'' + t_{jk}'') s_j'' (\frac{d}{2} + t_{ik}'' + t_{jk}'') s_k''} \\
&\times \frac{(\frac{d-2}{2})_{(t_{ij}'+t_{ik}'+t_{ij}''+t_{ik}'')/2} (\frac{d-2}{2})_{(t_{ij}^v+t_{jk}^v+t_{ij}''+t_{jk}'')/2} (\frac{d-2}{2})_{(t_{ik}^v+t_{jk}^v+t_{ik}''+t_{jk}'')/2}}{\prod [(\frac{d-2}{2})_{t_{ij}''+t_{ik}''}]^2 (\frac{d-2}{2} + t_{ik}'' + t_{jk}'') t_{ij}'''} \quad (23)
\end{aligned}$$

We notice that the symmetry $l \leftrightarrow l'$ is manifest, while this is not the case for $i \leftrightarrow j \leftrightarrow k$: we can explain this noticing that the summation \sum_l already breaks this symmetry. This expression can be written as $\phi = M^T D M$, with M triangular and D diagonal. This allows us to calculate the determinant of ϕ :

$$\det(\phi) = \prod_l \frac{\prod s_i! t_{ij}! (d/2 - t_{ij} - 1)_{t_{ik}+t_{jk}}}{(d-2)_{\Sigma t_{ij}} (d/2)_{\Sigma t_{ij}} \prod N_i! (d/2 + t_{ik} + t_{jk})_{s_i} (d/2 + t_{ik} + t_{jk})_{t_{ij}}} \quad (24)$$

We have tested the correctness of expressions (23) and (24) in a large number of cases by comparing with the inverse of the χ -matrix generated by recurrence relations (the number of tensor indices had to be small enough to allow the explicit inversion done by `Mathematica` without causing a hang-up).

5 Summary and conclusions

We have two mayor results to report. Firstly there are the general recurrence relations for the generation of the contraction matrix which are as far as we know new in the literature. In itself this expression is quite useful: for small and most common matrices explicit inversion is easy, for larger matrices and

in all practical cases we are solely interested in a ε -expansion. Since every element of the contraction matrix is polynomial in d , we can easily perform the inversion of χ perturbatively in ε [2].

Secondly in the 3-loop case we succeed in constructing a symmetrical explicit expression for the decomposition matrix. Apart from the esthetical satisfaction of finding a non-recurrent solution it is also indispensable for the large matrix cases. On the other hand even for the every-day cases it is a fast and direct way to generate the decomposition matrix, either in the case of analytic expression as a function of d or as an ε -expansion.

Generalisation towards more than three loops seems to be non-trivial: we do not get a unique configuration in the four-loop case using the recurrence relations of section 3 (e.g. every loop has 2 tensor indices: then there are several cases with $s_i = 0$). So we need more than a simple extension of the tric used in [1]. Although we are a bit closer towards the solution of the L -loop problem, a general solution for this intriguingly simply-looking problem is still lacking.

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